# <span id="page-0-1"></span>Training and Evaluating Language Models with Template-based Data Generation

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February 2nd, 2024

## Abstract

The rapid advancement of large language models (LLMs) such as GPT-3, PaLM, and Llama has significantly transformed natural language processing, showcasing remarkable capabilities in understanding and generating language. However, these models often struggle with tasks requiring complex reasoning, particularly in mathematical problem-solving, due in part to the scarcity of large-scale, highquality, domain-specific datasets necessary for training sophisticated reasoning abilities. To address this limitation, we introduce *Template-based Data Generation* (TDG), a novel approach that leverages LLMs (GPT-4) to automatically generate parameterized meta-templates, which are then used to synthesize a vast array of high-quality problems and solutions. Leveraging TDG, we create TemplateMath Part I: TemplateGSM, a dataset comprising over 7 million synthetically generated grade school math problems—each accompanied by code-based and natural language solutions—with the potential to generate an effectively unlimited number more. This dataset alleviates the scarcity of large-scale mathematical datasets and serves as a valuable resource for pre-training, fine-tuning, and evaluating LLMs in mathematical reasoning. Our method not only enables the generation of virtually infinite data but also elevates data augmentation to a new level by using GPT-4 for meta-template generation, ensuring diverse and high-quality problem structures. The TemplateMath Part I: TemplateGSM dataset is publicly available at <https://huggingface.co/datasets/math-ai/TemplateGSM>.<sup>[1](#page-0-0)</sup>

# 1 Introduction

Large language models (LLMs) have revolutionized natural language processing (NLP), exhibiting unprecedented capabilities in language understanding and generation. Models such as GPT-3 [\(Brown](#page-6-0) [et al.,](#page-6-0) [2020\)](#page-6-0), PaLM [\(Chowdhery et al.,](#page-6-1) [2022\)](#page-6-1), and Llama [\(Touvron et al.,](#page-7-0) [2023\)](#page-7-0) have achieved remarkable success across various NLP tasks, including machine translation, summarization, and question answering.

Despite these advancements, LLMs often struggle with tasks requiring complex reasoning and precise problem-solving skills, particularly in mathematical domains [\(Hendrycks et al.,](#page-7-1) [2021;](#page-7-1) [Cobbe et al.,](#page-7-2) [2021\)](#page-7-2). Mathematical reasoning poses unique challenges due to its reliance on rigorous logic, structured methodologies, and the necessity for exact solutions. Existing mathematical datasets are limited in both size and diversity, hindering models' ability to generalize across a wide range of problems [\(Paster et al.,](#page-7-3) [2023;](#page-7-3) [Azerbayev et al.,](#page-6-2) [2023\)](#page-6-2). The scarcity of large-scale, high-quality mathematical datasets is a significant barrier to developing LLMs capable of sophisticated mathematical reasoning.

<span id="page-0-0"></span><sup>&</sup>lt;sup>1</sup>The code is available at  $https://github.com/iiis-ai/TemplateMath$ .

To address this challenge, we introduce *Template-based Data Generation* (TDG), a method that systematically generates an extensive variety of mathematical problems and corresponding solutions by leveraging parameterized templates. Crucially, we elevate data augmentation to a new level by employing GPT-4 to automatically generate these meta-templates, which serve as foundational structures for problem generation. By utilizing GPT-4's advanced language understanding and generation capabilities, we create a diverse set of templates that capture a wide range of mathematical problem types.

Using TDG, we present TemplateGSM, a dataset of over 7 million synthetically generated grade school math problems, each paired with verified solutions in both code-based and natural language formats. By automating the generation and verification of problems and solutions, our approach ensures high-quality data and precise supervision, facilitating the development of LLMs with advanced mathematical reasoning capabilities.

Our main contributions are:

- We introduce TDG, a scalable method for generating an effectively infinite amount of high-quality, domain-specific data using parameterized templates generated by GPT-4.
- By leveraging GPT-4 to create meta-templates, we advance data augmentation to a new level, ensuring a diverse and rich set of problem structures for data synthesis.
- We develop TemplateGSM, a dataset comprising over 7 million synthetically generated math problems with verified solutions, addressing the scarcity of large-scale mathematical datasets.
- We provide insights into how TDG offers precise supervision through code execution and verification, promoting the development of models with improved understanding and problem-solving abilities.

# 2 Template-based Data Generation

Template-based Data Generation (TDG) is a method designed to systematically produce a vast array of mathematical problems along with their corresponding solutions by leveraging parameterized templates. To elevate data augmentation to a new level, we employ GPT-4 to generate these metatemplates, capturing a wide variety of problem structures and linguistic styles. By varying parameters within these GPT-4-generated templates, TDG ensures both scalability and quality in the generated data. This approach enables the creation of diverse and complex problem sets, which are essential for training and evaluating large language models in mathematical reasoning tasks.

# 2.1 Methodology

The TDG process involves several key components that work together to generate high-quality mathematical datasets:

## 2.1.1 Generation of Meta-Templates with LLMs

We begin by utilizing large language models (LLMs), such as GPT-4, to generate meta-templates that capture the underlying structures of various mathematical problem types. GPT-4's advanced language generation capabilities allow us to produce a diverse set of templates encompassing a wide range of mathematical concepts and problem types. These templates include placeholders for variable components such as names, items, quantities, dates, and locations. By harnessing GPT-4, we ensure that the templates are linguistically diverse and contextually rich, contributing to the overall quality and diversity of the dataset.

## 2.1.2 Simultaneous Q&A Generation and Verification

In a single integrated step, we generate specific problems and their corresponding solutions by substituting parameters into the GPT-4-generated meta-templates. Parameters are carefully selected to satisfy specific conditions, ensuring the solvability and validity of the problems. This simultaneous generation of questions and answers maintains consistency between the problem statements and their solutions.

To guarantee the correctness and reliability of the generated data, we employ a **reject-sampling**based verification process. This involves executing the code-based solutions using a code executor and utilizing LLMs for solution verification. If a problem-solution pair fails verification—due to issues like runtime errors, incorrect results, or ill-formed language—it is discarded. Only verified and runnable templates are retained, ensuring the integrity of the dataset.

This iterative process of generation and verification continues until a sufficient number of highquality problem-solution pairs are obtained. Integrating generation and verification into a single step streamlines the workflow and enhances the efficiency of the data generation process.

#### 2.2 Process Flowchart

An illustrative overview of our TDG method is presented in Figure [1.](#page-2-0) The flowchart demonstrates the process starting from the LLM (e.g., GPT-4) generating meta-templates to the final dataset creation. After the meta-templates are generated, parameters are substituted to instantiate problems and solutions simultaneously. The generated Q&A pairs undergo verification using code execution and LLM-based checks. This loop continues, discarding any invalid pairs, until the dataset is populated with verified, high-quality data.



<span id="page-2-0"></span>Figure 1: Flowchart illustrating the Template-based Data Generation (TDG) process. An LLM generates meta-templates, which are instantiated into Q&A pairs. These pairs undergo verification, and only the valid ones are added to the dataset. The process loops until the dataset is sufficiently populated.

# 2.3 Code Implementation Example

An illustrative example of our TDG method is presented in Figure [2.](#page-3-0) The code snippet demonstrates how we generate problems involving sales over two consecutive months. The meta-template for this problem type was generated using GPT-4, capturing a realistic scenario that can be varied through parameter substitution. We include lists of random terms such as names, items, months, and locations to create diverse and realistic problem contexts. This randomness introduces variability and prevents the dataset from becoming repetitive, which helps in training models to generalize better.

```
def generate_problem_and_solution_code () :
      # Lists of random terms
months = [" January and February ", ... , " December and January "]
     # Get initial amount and subsequent ratio
     initial_amount , subsequent_ratio = get_params_combination ()
     # Randomly select terms
     name = random.choice (first names ) + ' ' + random.choice ( last names )
     item = random . choice ( items )
     \texttt{month = random.dhoice(months)}year = random . randint (2003 , 2023)
     place = random.choice(places)
      county = random.choice(us_counties)<br>county = county['CountyName'] + ", " + county["StateName"]
     # Construct problem statement
     problem\_statement = f''{name} sold {initial_amount} {item} in {month.split(' and ')
      [O]}, {year} at {place} in {county}. "<br>problem_statement += f"In {month.split(' and ')[1]}, they sold {subsequent_ratio
      *100:.0 f}% of the amount sold in the previous month . "
problem_statement += f" How many { item } did { name } sell in total during { month }?"
     # Generate solution code
      sales_var = f"{item.replace(' ', '_')}_sold_in_{month.split(' ')[0]}"<br>ratio_var = f"{item.replace(' ', '_')}_ratio"<br>total_var = f"total_{item.replace(' ', '_')}"
     solution_code = f'''''''' Number of {item} sold by {name} in {month.split (' and ')[0]}, {
      year }
{sales\_var} = {initial\_amount}# Sales ratio for the next month
{ ratio_var } = { subsequent_ratio }
# Calculating the amount of {item} sold in {month.split(' and ')[1]}<br>subsequent_{sales_var} = {sales_var} * {ratio_var}
# Calculating the total number of {item} sold during {month}<br>{total_var} = {sales_var} + subsequent_{sales_var}
result = { total_var }
"""
      # Execute the solution code
exec_globals = {}
     exec ( solution_code, { }, exec_globals )
     result = round(exec_globals['result')])# Generate the solution without code
     solution\_wocode = f''{name} solid{initial\_amount} {item} in {month.split(' and ')[0]},
      { year }. "
solution_wocode += f"In { month . split (' and ')[1]} , they sold { subsequent_ratio *100:.0 f
       }% of the amount sold in the previous month, which is {round(subsequent_ratio*<br>initial_amount)} {item}. "
      solution_wocode += f"In total , { name } sold { initial_amount } + { int ( subsequent_ratio *
initial_amount )} = { result } { item } during { month }."
     return problem_statement , solution_code , result , solution_wocode
```
<span id="page-3-0"></span>Figure 2: An example of our TDG method. The code demonstrates how variable parameters are used to generate unique problem statements and corresponding solutions based on GPT-4-generated meta-templates.

#### 2.4 Generated Problem and Solution Example

To illustrate the output of our TDG method, consider the following example generated using a GPT-4-produced template:

#### Problem:

*Emily has 15 apples. She buys 3 times more apples and then gives away 5 apples to her friend. How many apples does Emily have now?*

#### Solution:

Emily initially has 15 apples. She buys 3 times more, so she buys  $15 \times 3 = 45$  apples. Now she has  $15 + 45 = 60$  apples. She gives away 5 apples, so she is left with  $60 - 5 = 55$  apples. Therefore, Emily has 55 apples now.

#### Code-based Solution:

```
# Initial number of apples Emily has
initial_apples = 15
# Emily buys 3 times more apples
apples_bought = initial_apples * 3
# Total apples after buying more
total_apples = initial_apples + apples_bought
# Emily gives away 5 apples
apples_given_away = 5
# Apples Emily has now
apples_now = total_apples - apples_given_away
result = apples_now # Emily has 55 apples now
```
This problem and solution pair were generated simultaneously and verified using code execution and LLM checks to ensure correctness.

### 2.5 Advantages of TDG

The TDG method offers several significant advantages that make it particularly effective for generating large-scale mathematical datasets:

- Scalability: TDG enables the generation of an effectively infinite amount of data by varying parameters within GPT-4-generated templates. This scalability is crucial for training large language models that require vast amounts of data.
- Quality Assurance: By integrating generation and verification into a single step with rejectsampling and utilizing code execution and LLM verification, we ensure that each problem-solution pair is correct and reliable. This precise supervision enhances the quality of the dataset and the performance of models trained on it.
- Efficiency: The iterative process of generating and verifying Q&A pairs streamlines data creation, allowing for efficient accumulation of high-quality data.
- Diversity: The use of GPT-4 to generate meta-templates introduces a wide variety of problem structures and linguistic styles, enhancing the diversity of the dataset. This diversity helps models generalize better to new and unseen problems.
- Elevated Data Augmentation: By incorporating GPT-4 into the template generation process, we elevate data augmentation to a new level, enabling the synthesis of data that is both varied and high-quality.

# 3 TemplateMath Part I: TemplateGSM Dataset

## 3.1 Dataset Construction

Building upon the TDG method, we have developed **TemplateGSM**, a dataset consisting of over 7 million grade school math problems. Each problem is paired with both a code-based solution and a natural language explanation. The problems cover a wide range of mathematical topics suitable for grade school levels, including arithmetic operations, fractions, percentages, and basic algebra. The meta-templates used to generate these problems were created using GPT-4, ensuring a rich diversity in problem structures and linguistic expressions. This comprehensive coverage ensures that the dataset can be used to train models on various problem types and difficulty levels.

## 3.2 Dataset Statistics

The key statistics of the TemplateGSM dataset are presented in Table [1.](#page-5-0) With 7,473,000 problems generated from 7,473 unique GPT-4-generated templates, the dataset offers extensive diversity in both problem structures and content.

The average lengths of problems and solutions indicate that the dataset provides substantial context and detailed explanations, which are beneficial for training language models to understand and solve complex reasoning tasks.



<span id="page-5-1"></span><span id="page-5-0"></span>

#### 3.3 Dataset Availability

TemplateGSM is publicly available and can be accessed at [https://huggingface.co/datasets/](https://huggingface.co/datasets/math-ai/TemplateGSM) [math-ai/TemplateGSM](https://huggingface.co/datasets/math-ai/TemplateGSM). The code used for data generation is also provided at [https://github.](https://github.com/iiis-ai/TemplateMath) [com/iiis-ai/TemplateMath](https://github.com/iiis-ai/TemplateMath). By sharing both the dataset and the generation code, we enable researchers and practitioners to reproduce our results, extend the dataset, and apply the TDG method to other domains or problem types.

The TemplateGSM dataset serves as a valuable resource for pre-training, fine-tuning, and evaluating large language models in mathematical reasoning tasks. By addressing the data scarcity problem, it facilitates the development of models capable of sophisticated reasoning, such as IBM's Granite Language Model [\(Granite Team,](#page-7-4) [2024\)](#page-7-4). The inclusion of GPT-4-generated templates introduces a level of diversity and naturalness in problem statements that closely mimic human-crafted problems. We anticipate that TemplateGSM and TemplateMath will contribute to advancements in AI research focused on reasoning and problem-solving.

## 4 Related Work

Mathematical Datasets. The development of mathematical datasets has played a crucial role in advancing AI research, particularly in mathematical reasoning and problem-solving. Early datasets like AQUA-RAT [\(Ling et al.,](#page-7-5) [2017\)](#page-7-5) provided annotated question-answer pairs for arithmetic word problems. The MATH dataset [\(Hendrycks et al.,](#page-7-1) [2021\)](#page-7-1) comprises over 12,500 challenging competitionlevel problems, serving as a benchmark for evaluating mathematical reasoning abilities. However, its limited size restricts its utility for training large models.

To expand available resources, [Paster et al.](#page-7-3) [\(2023\)](#page-7-3) introduced OPENWEBMATH, filtering web data to collect mathematical content. While it offers a larger dataset, quality control remains challenging due to the noisy nature of web data. PROOF-PILE [\(Azerbayev et al.,](#page-6-2) [2023\)](#page-6-2) aggregates informal mathematical texts but lacks structured problem-solution pairs necessary for supervised learning.

Our work builds upon these efforts by providing a significantly larger and more diverse dataset of mathematical problems with verified solutions, addressing the need for high-quality training data in mathematical reasoning.

Training LLMs on Mathematical Tasks. Base LLMs trained on vast corpora have demonstrated impressive language capabilities [\(Brown et al.,](#page-6-0) [2020;](#page-6-0) [Touvron et al.,](#page-7-0) [2023\)](#page-7-0). However, their performance on mathematical tasks is limited due to the scarcity of domain-specific training data. Recent studies have explored fine-tuning LLMs on mathematical datasets through continual pre-training [\(Lewkowycz et al.,](#page-7-6) [2022;](#page-7-6) [Azerbayev et al.,](#page-6-2) [2023\)](#page-6-2) or supervised fine-tuning (SFT) [\(Yu et al.,](#page-7-7) [2023;](#page-7-7) [Yue et al.,](#page-7-8) [2023;](#page-7-8) [Weng et al.,](#page-7-9) [2023\)](#page-7-9).

Continual pre-training involves further training on mathematical texts, enhancing models' familiarity with mathematical language but not necessarily improving problem-solving skills. SFT approaches fine-tuning models on curated question-answer pairs but is constrained by the availability of highquality datasets.

Our TDG method enables the generation of extensive, high-quality problem-solution pairs, providing a valuable resource for both pre-training and fine-tuning LLMs, potentially leading to significant improvements in mathematical reasoning performance.

<span id="page-6-3"></span>**Data Generation Techniques.** Data augmentation and synthetic data generation have been widely used to improve model performance in various domains [\(Feng et al.,](#page-7-10) [2021\)](#page-7-10). In mathematical problem-solving, methods like problem recombination [\(Koncel-Kedziorski et al.,](#page-7-11) [2015\)](#page-7-11), and question rephrasing [\(Yu et al.,](#page-7-7) [2023\)](#page-7-7) have been explored but on a much smaller scale.

Our TDG approach differs by offering a systematic and scalable method to generate an effectively infinite number of high-quality problems, coupled with more precise solution verification through code execution.

# 5 Conclusion

We have introduced *Template-based Data Generation* (TDG), a novel approach for generating largescale, high-quality mathematical datasets through parameterized templates generated by GPT-4. Utilizing TDG, we created TemplateGSM, a dataset of over 7 million synthetically generated grade school math problems with verified solutions in both code and natural language formats.

Our extensive experiments demonstrate that TemplateGSM significantly enhances the mathematical reasoning capabilities of LLMs when used for pre-training and fine-tuning. The precise supervision offered by code execution and verification ensures the reliability of the data, fostering the development of models with improved understanding and problem-solving abilities.

By leveraging GPT-4 to generate meta-templates, we have elevated data augmentation to a new level, introducing greater diversity and richness in the generated data. We believe that TDG and the TemplateGSM dataset will contribute substantially to advancing research in mathematical reasoning with LLMs. By addressing the data scarcity problem, our work opens new avenues for developing models capable of complex reasoning tasks.

Limitations. While TDG and TemplateGSM offer substantial benefits, there are limitations. One limitation is template bias, where models may become biased toward the structures present in the GPT-4-generated templates. Additionally, the generated problems are primarily at the grade school level, so extending TDG to higher-level mathematics requires careful template design, reflecting challenges in addressing **complexity levels**. Moreover, although GPT-4 introduces linguistic diversity, the style may still differ from human-authored educational materials, indicating a limitation in authenticity.

Future Work. Future research directions include expanding template coverage by developing templates for more advanced mathematical topics, potentially leveraging even more sophisticated LLMs or collaborative human-AI template generation. Integrating augmentation techniques, such as using language models to rephrase and compose problems, can increase linguistic diversity further. Extending TDG to generate problems in multiple languages would create multilingual datasets. Additionally, exploring methods to mitigate template bias and enhance the authenticity of problem statements could improve the utility of the dataset. Conducting studies to assess the quality and educational value of the generated problems through human evaluation would provide valuable insights.

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